FREE VIBRATION ANALYSIS OF LAMINITED COMPOSITE PLATE USING NEW HIGHER ORDER SHEAR DEFORMATION PLATE THEORY

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ABSTRACT

In the present work a theoretical analysis depends on the new higher order element in shear deformation theory for simply supported cross-ply laminated plate is developed. The new displacement field of the middle surface expanded as a combination of exponential and trigonometric function of thickness coordinate with the transverse displacement taken to be constant through the thickness. The governing equations are derived using Hamilton's principle and solved using Navier solution method to obtain the natural frequency. The effect of many design parameters such as number of laminates, aspect ratio and thickness ratio on dynamic behavior of the laminated composite plate have been studied. The modal of the present work has been verified by comparing the results of shape functions with that obtained by other worker. Result shows the good agreement with 3D elasticity solution and that published by other researcher.

Key Words : Higher order shear deformation theory, composite laminated plate, free vibration analysis, natural frequency .

التحليل الديناميكي للصفيحة المركبة بأستخدام نظرية قص ذات رتبة عالية جديدة

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الخلاصة

في هذا البحث تم تطوير الحل النظري الذي يعتمد على نظرية قص جديدة ذات رتبة عالية للصفائح المركبة ذات الاسناد البسيط . مجال الازاحة الجديد للسطح الاوسط يتوسع لدمج الدالة الاسية و المثلثية و تكون الازاحة المستعرضة ثابتة خلال السمك. تم اشتقاق معادلات الحركة بأستخدام (Hamilton's principle) و تم حلها بأستخدام طريقة الحل (Navier) لأيجاد التردد الطبيعي. تأثير عدة عوامل تصميمية مثل عدد الطبقات ، نسبة الطول الى العرض، نسبة السمك تم دراستها. بينت النتائج موافقة جيدة مع الحل بأستخدام نظرية مرنة ثلاثية الابعاد و البحوث المنشورة لباحثين اخرين. النموذج الرياضي للعمل الحالي تم اثباته بالمقارنة مع نتائج الدوال للباحثين الاخرين.

الكلمات الرئيسية : نظرية القص ذات الرتبة العالية ، الالواح الطبقية المركبة ، تحليل الأهتزاز الحر ، التردد الطبيعي

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NOMENCLATURE :-

Symbol	**Discretion**	Units
a	Plate dimension in x-direction	m
b	Plate dimension in y-direction	m
h	Plate thickness	m
A_{ij} , B_{ij} ,D_{ij} ,E_{ij} F_{ij} ,H_{ij}	Extension, bending extension coupling, bending and additional stiffness	-
E_1 , E_2 ,E_3	Elastic modulus components	Gpa
G_{12} , G_{23} ,G_{13}	Shear modulus components	Gpa
N	Total number of plate layers	-
N_1 , N_2 ,N_6	In-plane force result	N/m
M_1 , M_2 ,M_6	Moment result per unit length	N.m/m
P_1 , P_2 ,P_6	Result force per unit length	N/m
Q_1 , Q_2	Transverse shear force result	N
K_1 , K_2	Transverse shear force result(HSDT)	N
Z	Distance from neutral axis	M
x,y,z	Cartesian coordinate system	M
z_k , z_{k+1}	Upper and lower lamia surface coordinates along z-direction	M
ε_{xx} , ε_{yy} ,ε_{xy}	Strain components	m/m
γ_{xz} , γ_{yz}	Transverse shear strain	m/m
ν_{12}	Poisson's ratio components	-
σ_{xx} , σ_{yy} ,σ_{xy} ,σ_{yz} σ_{xz}	Stress components	Gpa
θ	Fiber orientation angle	Degree
U_{mn} , V_{mn} ,W_{mn} ,$\phi^1{}_{mn}$ $\phi^2{}_{mn}$	Arbitrary constant	-
u(x,y)	Flexural displacement	-
v(x,y)	Flexural displacement	-
w(x, y)	Flexural displacement	-
α	$\frac{m\pi}{h}$	-
β	$\frac{n\pi}{h}$	-
C_{ij}	Stiffness matrix	-
1,2,3	Principal material coordinate system	-
Wn	non-dimensional natural frequency	Rad/sec

INTRODUCTION :-

Composite materials are so necessary in many engineering applications, as vehicles parts industry, aero structures industry and medical devices industry. With the wide use of composite plate in the modern industry, static and dynamic analysis of plate structure under different types of loads and different boundary condition become a main part in design procedure. In the past few years, many researchers resorted to the development of many theories to clearly predict the response of laminated plate composite material. It is necessary to know the theories of laminated composite plates, because it is not possible to provide accurate analysis without knowledge of theories. These theories can be classified in to three type's single layer theories, layer-wise theories and continuum based 3D elasticity theories **Pervez.et al.2010**. Many researchers had studied static and dynamic analysis of composite plate by using higher order shear deformation theory, and other researchers have studied the natural frequency of simply supported composite plate.**N. D. Phan.et al.1985.** developed Analysis of laminated composite plates using a higher-order shear deformation theory. A higher-order shear deformation theory used to analysis laminated anisotropic composite plates for deflections, stresses, natural frequencies and buckling loads. The theory accounts for parabolic distribution of the transverse shear stresses, and requires no shear correction coefficients. A displacement finite element model of the theory developed, and applications of the element to bending, Vibration and stability of laminated plates were discussed. The present solutions are compared with those obtained using the classical plate theory and the three-dimensional elasticity theory. **M.Rastgaar Aagaah.et al. 2006.** Studied Natural frequencies of laminated composite plates used third order shear deformation theory. Natural frequencies of square laminated composite plates for different supports at edges presented. Using a third order shear deformation theory of plates (TSDT), which was categorized in equivalent single layer theories (ESL), a new set of linear equations of motion for square multi-layered composite plates had been derived. Laminated plates were supposed to be either angle ply or cross-ply. Moreover, FEM was used to solved the equations and find the fundamental natural frequencies. Finally some results for plates with different combination of layers and supports are reported. The results are compared to the results. **Hiroyuki Matsunaga 2000.** Developed .Natural frequencies and buckling stresses of cross-ply laminated composite plates are analyzed by taking into account the effects of shear deformation, thickness change and rotatory inertia. By using the method of power series expansion of displacement components, a set of fundamental dynamic equations of a two-dimensional higher-order theory for thick rectangular laminates subjected to in-plane stresses is derived through Hamilton's principle. Several sets of truncated approximate theories are applied to solve the eigenvalue problems of a simply supported thick laminated plate. In order to assure the accuracy of the present theory, convergence properties of the lowest natural frequency and buckling stress are examined in detail. Numerical results are compared with those of the published existing theories and FEM solutions. The modal displacement and stress distributions in the thickness direction are obtained and plotted in figures. It is noticed that the present global higher-order approximate theories can predict the natural frequencies, buckling stresses and stresses of thick multilayered composite laminates as accurately as three dimensional solutions. Ration and stability of cross-ply laminated composite plates according to a global higher-order plate theory. **Akavci.et al.2003.** Presented buckling and free vibration analysis of laminated composite plate by using two new hyperbolic shear-deformation theories. Two new hyperbolic displacement models, HPSDT1 and HPSDT2, are used for the buckling and free vibration analysis of simply supported orthotropic laminated composite plates. The models contain hyperbolic expressions to account for the parabolic distributions of transverse shear stresses and to satisfy the zero shear-stress conditions at the top and bottom surfaces of the plates. The equation of motion for thick laminated rectangular plates subjected to in-plane loads is deduced through the use of Hamilton's principle. Closed-form solutions are obtained by using the Navier technique, and then the buckling loads and the fundamental frequencies are found by solving eigenvalue problems. **Song Xiang.et al. 2009.** Presented Natural frequencies of generally laminated composite plates using the Gaussian radial basis function and first-order shear deformation theory. The Gaussian radial basis functions and first-order shear deformation theory are presented to calculate the natural frequencies of generally laminated composite plates. Several numerical examples are used to show convergence and accuracy of the present method. The results of the present paper are in good agreement with the results already reported in the literature, which demonstrate the high numerical accuracy of the Gaussian radial basis function for vibration analysis of generally laminated composite plates. **Mantari.et al.** studied Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher-order shear deformation theory. A new higher order shear deformation theory for elastic composite/sandwich plates and shells is developed. The new displacement field depends on a parameter ‘‘m’’, whose value is determined so as to give results closest to the 3D elasticity bending solutions. The present theory accounts for an approximately parabolic distribution of the transverse shear strains through the shell thickness and tangential stress-free boundary conditions on the shell boundary surface. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are solved using Navier-type, closed form solutions. Shells and plates are subjected to bi-sinusoidal, distributed and point loads. Results are provided for thick to thin as well as shallow and deep shells. **Adnan Naji.et al. 2012.** Developed free vibration analysis of laminated composite plate using HOST 12. The present works an application of a Higher Order Shear Deformation Theory (HOST 12) to problem of free vibration of simply supported symmetric and anti-symmetric angle-ply composite laminated plates. The theoretical model HOST12 presented incorporates laminate deformations which account for the effects of transverse shear deformation, transverse normal strain/stress and a nonlinear variation of in-plane displacements with respect to the thickness coordinate – thus modeling the warping of transverse cross-sections more accurately and eliminating the need for shear correction coefficients. Solutions are obtained in closed-form using Navier’s technique by solving the eigenvalue equation. Plates with varying number of layers, degrees of anisotropy and slenderness ratios are considered for analysis. **Huu.et al.2013.** Various efficient higher-order shear deformation theories are presented for bending and free vibration analyses of functionally graded plates. The displacement fields of the present theories are chosen based on cubic, sinusoidal, hyperbolic, and exponential variations in the in-plane displacements through the thickness of the plate. By dividing the transverse displacement into the bending and shear parts and making further assumptions, the number of unknowns and equations of motion of the present theories is reduced and hence makes them simple to use. Equations of motion are derived from Hamilton’s principle. Analytical solutions for deflections, stresses, and frequencies are obtained for simply supported rectangular plates. The accuracy of the present theories is verified by comparing the obtained results with the exact three-dimensional (3D) and quasi-3D solutions and those predicted by higher-order shear deformation theories. **Atteshamuddin S. Sayyad.et al. 2015.** Studied Bending, Vibration and Buckling of Laminated Composite Plates Using a Simple Four Variable Plate Theory. A simple trigonometric shear deformation theory is applied for the bending, buckling and free vibration of cross ply laminated composite plates. The theory involves four unknown variables which are five in first order shear deformation theory or any other higher order theories. The in-plane displacement field uses sinusoidal function in terms of thickness co-ordinate to include the shear deformation effect. The transverse displacement includes bending and shear components. Satisfies the zero shear stress conditions at top and bottom surfaces of plates without using shear correction factor. Equations of motion associated with the present theory are obtained using the dynamic version of virtual work principle. A closed form solution is obtained using double trigonometric series suggested by Navier.

In the present work, a new higher order displacement field in which the displacement of the middle surface expanded as a combination of exponential and trigonometric functions of the thickness coordinate and the transverse displacement taken to be constant through the thickness, is proposed. Necessary equilibrium equations and boundary conditions are derived by employing the principle of virtual work. The theory accounts for adequate distribution of the transverse shear strains through the plate thickness and the tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. Exact solutions for natural frequency of simply supported plates are presented.

THEORETICAL ANALYSIS :-

Displacement Field:

In the present work, a new higher order displacement field in which the displacement of the middle surface expanded as a combination of exponential trigonometric function of the thickness coordinate and the transverse displacement taken to be constant through the thickness is developed. The displacement field of the new higher order theory of laminated composite plate is: **J.N.Raddy 2003**

$$u(x,y,z) = u(x,y) - z\left(\frac{\partial w}{\partial x}\right) + f(z)\theta_1(x,y) \quad (1\text{-a})$$

$$v(x,y,z) = v(x,y) - z\left(\frac{\partial w}{\partial y}\right) + f(z)\theta_2(x,y) \quad (1\text{-b})$$

$$w(x,y,z) = w(x,y) \quad (1\text{-c})$$

Where:
$u(x,y), v(x,y), w(x,y), \theta_1(x,y), \theta_2(x,y)$ Are the five unknown functions of middle surface of the plate as shown in the figure(1) while $f(z)$ represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness.

The shape function derived by different researchers are given in table 1, actually the present modeling is a combination of exponential functions and polynomial as shown in figure(2). With the same Reddy and Liu and generalized procedure developed by Sadatos and free boundary conditions at the top and bottom surfaces of the plate. The new displacement field presented in this paper is:

$$u(x,y,z) = u(x,y) + z\left(\frac{m\pi}{h}\theta_1 - \frac{\partial w}{\partial x}\right) + \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\theta_1 \quad (2\text{-a})$$

$$v(x,y,z) = v(x,y) + z\left(\frac{m\pi}{h}\theta_2 - \frac{\partial w}{\partial y}\right) + \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\theta_2 \quad (2\text{-b})$$

$$w(x,y,z) = w_0 \quad (2\text{-c})$$

where the new function used in present work is:

$$\mathbf{f(z) = \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}} + yz} \quad (3)$$

$$y = \frac{\pi m}{h}, \; m = \text{constant}$$

For small strains, the strain-displacement relations take the form:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \quad (4\text{-a})$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} \quad (4\text{-b})$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \tag{4-c}$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\gamma_{xy} \tag{4-d}$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}\gamma_{xz} \tag{4-e}$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz} \tag{4-f}$$

The strain associated with the displacement field by substituting equations (2a-c) into equations (4a-e) to give:

$$\varepsilon_{xx} = \varepsilon^0_{xx} + z\varepsilon^1_{xx} + \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\ \varepsilon^2_{xx} \tag{5-a}$$

$$\varepsilon_{yy} = \varepsilon^0_{yy} + z\varepsilon^1_{yy} + \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\ \varepsilon^2_{yy} \tag{5-b}$$

$$\gamma_{xy} = \varepsilon^0_{xy} + z\varepsilon^1_{xy} + \sin\frac{\pi z}{h} e^{\frac{m\pi z}{h}}\ \varepsilon^2_{xy} \tag{5-c}$$

$$\gamma_{xz} = \varepsilon^0_{xz} + \left(m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}\right)\frac{\pi}{h} e^{\frac{m\pi z}{h}}\ \varepsilon^3_{xz} \tag{5-d}$$

$$\gamma_{yz} = \varepsilon^0_{yz} + \left(m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}\right)\frac{\pi}{h} e^{\frac{m\pi z}{h}}\ \varepsilon^3_{yz} \tag{5-e}$$

Where:

$$\begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{m\pi}{h}\frac{\partial \theta_1}{\partial x_1} - \frac{\partial^2 w}{\partial x^2} \\ \frac{m\pi}{h}\frac{\partial \theta_2}{\partial y} - \frac{\partial^2 w}{\partial y^2} \\ \frac{m\pi}{h}\frac{\partial \theta_2}{\partial x_1} + \frac{m\pi}{h}\frac{\partial \theta_2}{\partial y} - 2\frac{\partial^2 w}{\partial x \partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon^2_{xx} \\ \varepsilon^2_{yy} \\ \gamma^2_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \theta_1}{\partial x} \\ \frac{\partial \theta_2}{\partial y} \\ \frac{\partial \theta_2}{\partial x} + \frac{\partial \theta_1}{\partial y} \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma^0_{xz} \\ \gamma^0_{yz} \end{Bmatrix} = \begin{Bmatrix} m\frac{\pi}{h}\theta_1 \\ m\frac{\pi}{h}\theta_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma^3_{xz} \\ \gamma^3_{yz} \end{Bmatrix} = \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} \tag{6}$$

Hamilton's principles:

The equation of motion of the new higher order theory will be derived using the dynamic version of the principle of virtual displacements: **Raddy, 2003**

$$0 = \int_0^t \delta U + \delta V - \delta K \tag{7}$$

The virtual strain energy δU is:

$$\delta U=[\int_{-\frac{h}{2}}^{\frac{h}{2}}\{\int_{\Omega}^{k} \sigma_{xx}\delta \varepsilon_{xx}^{k} + \sigma_{yy}\delta\varepsilon_{yy}^{k} + \sigma_{xy}\delta\varepsilon_{xy}^{k} + \sigma_{yz}\delta\varepsilon_{yz}^{k} + \sigma_{xz}\delta\varepsilon_{xz}^{k}]\,\partial x\partial y\}\partial z] = 0 \qquad (8)$$

$$\delta U = \int (N_1\delta\varepsilon_{xx}^{0} + M_1\delta\varepsilon_{xx}^{1} + P_1\delta\varepsilon_{xx}^{2} + N_2\delta\varepsilon_{yy}^{0} + M_2\delta\varepsilon_{yy}^{1} + P_2\delta\varepsilon_{yy}^{2} + N_6\delta\varepsilon_{Xy}^{0} + M_6\delta\varepsilon_{Xy}^{1} + P_6\delta\varepsilon_{Xy}^{2} + Q_2\delta\varepsilon_{yz}^{0} + k_2\delta\varepsilon_{yz}^{3} + Q_1\delta\varepsilon_{xz}^{0} + k_1\delta\varepsilon_{xz}^{3} -)\partial x\partial y = 0 \qquad (9)$$

Where:
(N_i,M_i,P_i,Q_i and K_i) are the result of the following integration:

$$(N_i, M_i, P_i) = \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\sigma_i^k\left(1\,,z,\sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}}\right)dz \qquad (i = 1,2,6)$$

$$(Q_1, K1)= \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\sigma_5^k\left(1,\frac{\pi}{h}(m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h})e^{\frac{m\pi z}{h}}\right)dz$$

$$(Q_2, K2)= \sum_{k=1}^{N}\int_{z^{k-1}}^{z^k}\sigma_4^k\left(1,\frac{\pi}{h}(m * \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h})e^{\frac{m\pi z}{h}}\right)dz$$

The virtual strains are known in terms of virtual displacement in equation (5) and then substituting the virtual strain into equation (9) and in integrating by parts to relative the virtual displacement (δu , δv , δw) in range of any differentiation, then we get:

$$0 = -\int[\frac{\partial N_1}{\partial x}\delta u + \frac{m\pi}{h}\frac{\partial M_1}{\partial x}\delta\theta_1 - \frac{\partial^2 M_1}{\partial x^2}\delta w + \frac{\partial P_1}{\partial x}\delta\theta_1 + \frac{\partial N_2}{\partial y}\delta v + \frac{m\pi}{h}\frac{\partial M_2}{\partial y}\delta\theta_2 - \frac{\partial^2 M_2}{\partial y^2}\delta w + \frac{\partial P_2}{\partial y}\delta\theta_2 + \frac{\partial N_6}{\partial y}\delta u + \frac{\partial N_6}{\partial x}\delta v + \frac{m\pi}{h}\frac{\partial M_6}{\partial y}\delta\theta_1 + \frac{m\pi}{h}\frac{\partial M_6}{\partial x}\delta\theta_2 + 2\frac{\partial^2 M_6}{\partial x\partial y}\delta w + \frac{\partial P_6}{\partial y}\delta\theta_1 + \frac{\partial P_6}{\partial x}\delta\theta_2 - \frac{m\pi}{h}Q_1\delta\theta_1 - \frac{m\pi}{h}Q_2 - K_1\delta\theta_1 - K_2\delta\theta_2]\,\partial x\partial y = 0 \qquad (10)$$

The virtual work done by applied forces δv is:

$$\delta V = -\int_{\Omega} q\,\delta w\,dxdy \qquad (11)$$

$$\delta K = \iint_{-\frac{h}{2}}^{\frac{h}{2}}\rho\{[u^{\cdot} + z\left(\frac{m\pi}{h}\theta_1^{\cdot} - \frac{\partial w^{\cdot}}{\partial x}\right) + f(z)\theta_1^{\cdot}]\left[\delta u^{\cdot} + z\left(\frac{m\pi}{h}\delta\theta_1^{\cdot} - \frac{\partial w^{\cdot}}{\partial x}\right) + f(z)\delta\theta_1^{\cdot}\right] + \left[v^{\cdot} + z\left(\frac{m\pi}{h}\theta_2^{\cdot} - \frac{\partial w^{\cdot}}{\partial y}\right) + f(z)\theta_2^{\cdot}\right] * \left[\delta v^{\cdot} + z\left(\frac{m\pi}{h}\delta\theta_2^{\cdot} - \frac{\partial w^{\cdot}}{\partial y}\right) + f(z)\delta\theta_2^{\cdot}\right] + w\delta w^{\cdot}\}dv$$

$$\delta K = \Big[\left(I_1u^{\cdot} + I_2\frac{m\pi}{h}\theta_1^{\cdot} - I_2\frac{\partial w^{\cdot}}{\partial x} + I_4\theta_1^{\cdot}\right)\delta u^{\cdot} + \left(I_2u^{\cdot} + I_3\frac{m\pi}{h}\theta_1^{\cdot} - I_3\frac{\partial w^{\cdot}}{\partial x} + I_5\theta_1^{\cdot}\right)\frac{m\pi}{h}\delta\theta_1^{\cdot} - \left(I_2u^{\cdot} + I_3\frac{m\pi}{h}\theta_1^{\cdot} - I_3\frac{\partial w^{\cdot}}{\partial x} + I_5\theta_1^{\cdot}\right)\frac{\delta\,\partial w^{\cdot}}{\partial x} + \left(I_4u^{\cdot} + I_5\frac{m\pi}{h}\theta_1^{\cdot} - I_5\frac{\partial w^{\cdot}}{\partial x} + I_6\theta_1^{\cdot}\right)\theta_1^{\cdot} + \left(I_1v^{\cdot} + I_2\frac{m\pi}{h}\theta_2^{\cdot} - I_2\frac{\partial w^{\cdot}}{\partial y} + I_4\theta_2^{\cdot}\right)\delta v^{\cdot} + \left(I_2v^{\cdot} + I_3\frac{m\pi}{h}\theta_2^{\cdot} - I_3\frac{\partial w^{\cdot}}{\partial y} + I_5\theta_2^{\cdot}\right)\frac{m\pi}{h}\theta_2^{\cdot} - \left(I_2v^{\cdot} + I_3\frac{m\pi}{h}\theta_2^{\cdot} - I_3\frac{\partial w^{\cdot}}{\partial y} + I_5\theta_2^{\cdot}\right)\frac{\delta\,\partial w^{\cdot}}{\partial y} + \left(I_4v^{\cdot} + I_5\frac{m\pi}{h}\theta_2^{\cdot} - I_5\frac{\partial w^{\cdot}}{\partial y} + I_6\theta_2^{\cdot}\right)\theta_2^{\cdot} + I_1w\delta w^{\cdot}\Big]dxdy \qquad (12)$$

Where: $I_1, I_2, I_3, I_4, I_5, I_6$ are the moment of inertia .

Equation of motion

The Euler-Lagrange is obtained by substituting Eq.(8 – 11) into Eq.(7) and then setting the coefficient of (δu , δv, δw , $\delta\Theta_1$, $\delta\Theta_2$) over Ω_0 of Eq.(7) to zero separately, this give five equations of motion as follows:

$$\delta u \rightarrow \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = I_1u^{\cdot\cdot} + I_2\frac{m\pi}{h}\theta_1^{\cdot\cdot} - I_2\frac{\partial w^{\cdot\cdot}}{\partial x} + I_4\theta_1^{\cdot\cdot}$$

$$\delta v \rightarrow \frac{\partial N_2}{\partial y} + \frac{\partial N_6}{\partial x} = I_1v^{\cdot\cdot} + I_2\frac{m\pi}{h}\theta_2^{\cdot\cdot} - I_2\frac{\partial w^{\cdot\cdot}}{\partial y} + I_4\theta_2^{\cdot\cdot}$$

$\delta w \rightarrow$

$$\frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + p = I_1 \ddot{w} - \left(I_2 \frac{\partial \ddot{u}}{\partial x} + I_3 \frac{m\pi}{h}\frac{\partial \ddot{\theta}_1}{\partial x} - I_3 \frac{\partial \ddot{w}^2}{\partial x^2} + I_5 \frac{\partial \ddot{\theta}_1}{\partial x}\right) - \left(I_2 \frac{\partial \ddot{v}}{\partial y} + I_3 \frac{m\pi}{h}\frac{\partial \ddot{\theta}_2}{\partial y} - I_3 \frac{\partial \ddot{w}^2}{\partial y^2} + I_5 \frac{\partial \ddot{\theta}_2}{\partial y}\right)$$

$\delta \Theta_1 \rightarrow$

$$\frac{m\pi}{h}\frac{\partial M_1}{\partial x} + \frac{m\pi}{h}\frac{\partial M_6}{\partial y} + \frac{\partial P_1}{\partial x} + \frac{\partial P_6}{\partial y} - \frac{m\pi}{h}Q_1 - K_1 = \left(I_2 \ddot{u} + I_3 \frac{m\pi}{h}\ddot{\theta}_1 - I_3 \frac{\partial \ddot{w}}{\partial x} + I_5 \ddot{\theta}_1\right)\frac{m\pi}{h} + \left(I_4 \ddot{u} + I_5 \frac{m\pi}{h}\ddot{\theta}_1 - I_5 \frac{\partial \ddot{w}}{\partial x} + I_6 \ddot{\theta}_1\right)$$

$\delta \Theta_2 \rightarrow$

$$\frac{m\pi}{h}\frac{\partial M_2}{\partial y} + \frac{m\pi}{h}\frac{\partial M_6}{\partial x} + \frac{\partial P_2}{\partial y} + \frac{\partial P_6}{\partial x} - \frac{m\pi}{h}Q_2 - K_2 = \left(I_2 \ddot{v} + I_3 \frac{m\pi}{h}\ddot{\theta}_2 - I_3 \frac{\ddot{w}}{\partial y} + I_5 \ddot{\theta}_2\right)\frac{m\pi}{h} + \left(I_4 \ddot{v} + I_5 \frac{m\pi}{h}\ddot{\theta}_2 - I_5 \frac{\ddot{w}}{\partial y} + I_6 \ddot{\theta}_2\right)$$

(13a-f)

The result forces are given by: **Raddy, 2003**

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} z\, dz$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} f(z)\, dz$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \sum_{k=1}^{n} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} \partial z$$

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \sum_{k=1}^{n} \begin{Bmatrix} \sigma_5 \\ \sigma_4 \end{Bmatrix} f(z)\, \partial z \qquad (14a\text{-}f)$$

The plane stress reduced stiffness Q_{ij} is:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{11} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \qquad (15)$$

From the constitutive relation of the k^{th} lamina the transformed stress-strain relation of an orthotropic lamina in a plane state of stress are:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \qquad (16)$$

The force results are related to the strains by the relations:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} B_{11} B_{12} B_{16} \\ B_{12} B_{22} B_{26} \\ B_{16} B_{26} B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} D_{11} D_{12} D_{16} \\ D_{12} D_{22} D_{26} \\ D_{16} D_{26} D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} F_{11} F_{12} F_{16} \\ F_{12} F_{22} F_{26} \\ F_{16} F_{26} F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_6 \end{Bmatrix} = \begin{bmatrix} E_{11} E_{12} E_{16} \\ E_{12} E_{22} E_{26} \\ E_{16} E_{26} E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + \begin{bmatrix} F_{11} F_{12} F_{16} \\ F_{12} F_{22} F_{26} \\ F_{16} F_{26} F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + \begin{bmatrix} H_{11} H_{12} H_{16} \\ H_{12} H_{22} H_{26} \\ H_{16} H_{26} H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} A_{55} & A_{54} \\ A_{45} & A_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{Bmatrix} + \begin{bmatrix} J_{55} & J_{54} \\ J_{45} & J_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz}^3 \\ \varepsilon_{yz}^3 \end{Bmatrix}$$

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{bmatrix} J_{55} & J_{54} \\ J_{45} & J_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz}^0 \\ \varepsilon_{yz}^0 \end{Bmatrix} + \begin{bmatrix} L_{55} & L_{54} \\ L_{45} & L_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xz}^3 \\ \varepsilon_{yz}^3 \end{Bmatrix} \qquad (17a\text{-}f)$$

Where:

$$A_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\, dz \qquad i = (1,2,4,5,6)$$

$$B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}(z, z^2, \sin(\frac{\pi z}{h}) e^{\frac{m\pi z}{h}}, \sin(\frac{\pi z}{h}) e^{\frac{m\pi z}{h}} z, \sin^2(\frac{\pi z}{h})\, e^{\frac{2m\pi z}{h}} \qquad i = (1,2,6)$$

$$I_1, I_2, I_3, I_4, I_5, I_6 = \int_{\frac{-h}{2}}^{\frac{h}{2}} \rho(1, z, z^2, f(z), f(z)z, [f(z)]^2) dz$$

$$J_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \frac{\pi}{h} e^{\frac{m\pi z}{h}} (m \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}) dz$$

$$L_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \left(\frac{\pi}{h}\right)^2 e^{\frac{m\pi z}{h}} \left(m \sin\frac{\pi z}{h} + \cos\frac{\pi z}{h}\right)^2 dz \qquad i = (4,5) \qquad (18a\text{-}f)$$

Navier's Solution

In Navier's method the generalized displacements are expanded in a double trigonometric series in terms of unknown parameters. The choice of the function in the series is restricted to those which satisfy the boundary conditions of the problem as shown in **Fig.3**. Substitution of the displacement expansion into the rning equations should give a set of algebraic equation among the parameter of the expansion.Simply supported boundary conditions are satisfied by assuming the following form of displacements: **Raddy, 2003**

$$u(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} \cos(\alpha x) \sin(\beta y)$$
$$v(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} \sin(\alpha x) \cos(\beta y)$$
$$w(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin(\alpha x) \sin(\beta y)$$
$$\theta_1(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \theta_{1_{mn}} \cos(\alpha x) \sin(\beta y)$$
$$\theta_2(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \theta_{2_{mn}} \sin(\alpha x) \cos(\beta y) \qquad (19)$$

where:

$\alpha = \frac{m\pi}{h}$, $\beta = \frac{n\pi}{h}$, $(U_{mn}, V_{mn}, W_{mn}, \theta_{1_{mn}}, \theta_{2_{mn}})$ are arbitrary constants.

The Navier solution exists if the following stiffnesses are zero, $A_{16} = B_{16} = D_{16} = E_{16} = F_{16} = H_{16} = A_{26} = B_{26} = D_{26} = E_{26} = F_{26} = H_{26} = A_{45} = J_{45} = L_{45} = I_2 = 0$

Substituting equation (19) into the equations of motion (13), we get the following eigen value equations at any fixed values of m and n:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} \end{bmatrix} - w^2 \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{12} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{13} & m_{23} & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & m_{45} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \theta_{1mn} \\ \theta_{2mn} \end{Bmatrix} = 0 \qquad (20)$$

For nontrivial solutions of equation (20) the corresponding determinants must be equal to zero:

$|[C] - w^2[M]| = 0$

Where the stiffness element of k_{ij} are:

$C_{11} = A_{11}\alpha^2 - A_{66}\beta^2$

$C_{12} = A_{12}\alpha\,\beta - A_{66}\alpha\,\beta$

$C_{13} = B_{11}\alpha^3 + B_{12}\alpha\beta^2 + 2B_{66}\alpha\beta^2$

$C_{14} = -B_{11}\frac{m\pi}{h}\alpha^2 - E_{11}\alpha^2 - B_{66}\frac{m\pi}{h}\beta^2 - E_{66}\beta^2$

$C_{15} = -B_{12}\frac{m\pi}{h}\alpha\,\beta - E_{12}\alpha\,\beta - B_{66}\frac{m\pi}{h}\alpha\,\beta - E_{66}\alpha\,\beta$

$C_{22} = -A_{22}\beta^2 - A_{66}\alpha^2$

$C_{23} = B_{12}\alpha^2\beta + B_{22}\beta^3 + 2B_{66}\alpha^2\beta$

$C_{24} = -B_{12}\frac{m\pi}{h}\alpha\,\beta - E_{12}\alpha\,\beta - B_{66}\frac{m\pi}{h}\alpha\,\beta - E_{66}\alpha\,\beta$

$C_{25} = -E_{22}\beta^2 - B_{66}\frac{m\pi}{h}\alpha^2 - E_{66}\alpha^2 - B_{22}\frac{m\pi}{h}\beta^2$

$C_{33} = -D_{11}\alpha^4 - 2D_{12}\alpha^2\beta^2 - D_{22}\beta^4 - 4D_{66}\alpha^2\beta^2$

$C_{34} = D_{11}\frac{m\pi}{h}\alpha^3 + D_{12}\frac{m\pi}{h}\alpha\beta^2 + F_{12}\alpha\beta^2 + F_{11}\alpha^3 + 2D_{66}\frac{m\pi}{h}\alpha\,\beta^2 + 2F_{66}\alpha\,\beta^2$

$C_{35} = D_{12}\frac{m\pi}{h}\alpha^2\beta + F_{12}\alpha^2\beta + D_{22}\frac{m\pi}{h}\beta^3 + F_{22}\beta^3 + 2D_{66}\frac{m\pi}{h}\alpha^2\beta^2 + F_{66}\alpha^2\beta$

$C_{44} = -D_{11}\frac{m^2\pi^2}{h^2}\alpha^2 - 2F_{11}\frac{m\pi}{h}\alpha^2 - D_{66}\frac{m^2\pi^2}{h^2}\beta^2 - 2F_{66}\frac{m\pi}{h}\beta^2 - H_{11}\alpha^2 - B66\beta^2 - F_{66}\beta^2 - A_{55}\frac{m^2\pi^2}{h^2} - 2J_{55}\frac{m\pi}{h} - L_{55}$

$C_{45} = -D_{12}\frac{m^2\pi^2}{h^2}\alpha\,\beta - 2F_{12}\frac{m\pi}{h}\alpha\,\beta - D_{66}\frac{m^2\pi^2}{h^2}\alpha\,\beta - 2F_{66}\frac{m\pi}{h}\alpha\,\beta - H_{12}\alpha\,\beta - H_{66}\alpha\,\beta$

$C_{55} = -D_{22}\frac{m^2\pi^2}{h^2}\beta^2 - 2F_{22}\frac{m\pi}{h}\beta^2 - D_{66}\frac{m^2\pi^2}{h^2}\alpha^2 - 2F_{66}\frac{m\pi}{h}\alpha^2 - H_{22}\beta^2 - H_{66}\alpha^2 - A_{44}\frac{m^2\pi^2}{h^2} - J_{44}\frac{m\pi}{h} - J_{44} - L_{44}$

And mass elements :

$m_{11} = I_1$

$m_{13} = -I_2\alpha = m_{31}$

$m_{14} = I_4\alpha + I_2\frac{m\pi}{h}\alpha = m_{41}$

$m_{22} = I_1$

$m_{23} = -I_2\beta = m_{32}$

$m_{25} = I_4 + I_2\frac{m\pi}{h} = m_{52}$

$m_{33} = I_1 + I_3(\alpha^2 + \beta^2)$

$m_{34} = -I_5\alpha - I_3\frac{m\pi}{h}\alpha = m_{43}$

$m_{35} = -I_5\beta - I_3\frac{m\pi}{h}\beta = m_{53}$

$$m_{44} = I_3 \frac{m^2\pi^2}{h^2} + 2I_5 \frac{m\pi}{h} + I_6$$
$$m_{55} = I_3 \frac{m^2\pi^2}{h^2} + 2I_5 \frac{m\pi}{h} + I_6$$

The main computer programming has been built to carry out the analysis required for solving the equations of motion and determine the fundamental natural frequency of composite laminated simply supported plate using new higher order shear deformation plate theory. A computer code written in (Matlab R13).

RESULTS AND DISCUSION :-

The natural frequency for free vibration of composite laminated plate with different design parameters for simply supported boundary condition, is analyzed and solved using MATLAP programming. To examine the validly of the derived equation and performance of computer programming for free vibration of composite laminated simply supported plate, a comparison [3D elasticity &J.Raddy & J.L.Mantari] for two, three, four and six layers cross ply Laminated (0/90/0) and (0/90/90/0) and simply supported on all edge, while the mechanical properties of each layers are (E_1=40, E_2=1, E_3=1, ρ=1,v_{12}=v_{13}=0.25, v_{23}=0, G_{12}=G_{13}=0.6, G_{23}=0.5). Table(2) show the fundimantal frequancy for a four-layer symmetric laminate square plate (a/h=5), as a function of modulus ratios (E_1/ E_2). For the ratios of E1/E2 change from 5 to 40, the percentage error in predicting the natural frequencies using the present theory is less than1% in all the cases. The present theory is also in very close agreement with the other shear deformation theories, however FSDT results are slightly higher than the exact one. Table(3) shows the Non-dimensional fundamental frequencies of antisymmetric square laminated plate for various values of modules ratio and thickness ratio (a/h = 5) The results of the present theory and other theaories such as (Raddy, S.S. Akavci and Kant) are compared with the three dimentional elasticity results (3D). for all the laminate types considered at lower range of (E_1 / E_2) ratio equal to (3 and 10), the error in present theory is less compared to other theories. Whereas for the laminates at higher range of (E_1 / E_2) ratio equal to (20 to 40) the theory of kant give better accurate results in comparison to other theories.Table (4) shows the Non-dimensional fundamental frequencies of square simply supported laminated plate (a=b) a cross ply (0/90/0) and orthotropic ratio $\frac{E_1}{E_2} = 40$, which shows a good results when compared with other researches such as (S.Xiang & K.Ming and Raddy).Table 5 we consider a four layer [0/90/90/0] simply supported plate with the effect of width-to-thickness ratio. The normalized frequency $w^{-} = (wb^2)\left(\frac{\rho}{E_2}\right)^{1/2}$ is showen in table 5.the results show that HSDT solution a gree well with available results given in Zhen.et al. 2006, Wu et al. 1994, Matsunaga 2000, Cho et al. 1991. Figure(4) shows that the fundamental frequency various side-to-thickness ratio (a/h) of laminated cross ply (0/90) for the present work compared with TSDT which shows the same trend of curve. Figure(5) shows that the fundamental frequency various side-to-thickness ratio (a/h) of laminated cross ply (0/90/0/90/0/90) for the present work compared with TSDT which shows the trend of curve for present work closed to the trend of curve for TSDT.

CONCLUSIONS :-

A new higher order shear deformation theory of laminated composite plates is presented. The theory accounts for an adequate distribution of the transverse shear strains through the plate thickness and the tangential stress-free boundary conditions on the plate boundary surface, therefore a shear correction factor is not required. The governing equations and boundary conditions are derived by employing the principle of virtual work. These equations are solved via a Navier-type, closed form solution. The accuracy of the present code is ascertained by comparing it with various available results in the literature. The results show that the present model performs better than all the higher order shear deformation theories compared here for analyzing the dynamic behavior of multilayered composite plates .

Table 1: Different shear shape strain functions.

Modal	f(z)function
Touratier 1991	$f(z) = \frac{h}{\pi}\sin\frac{\pi z}{h}$
Karma 2003	$f(z) = ze^{-2(z/h)^2}$
Mantari 2012	$f(z) = \sin\frac{\pi z}{h}e^{\cos\frac{m\pi z}{h}} + yz$
Present	$f(z) = \sin\frac{\pi z}{h}e^{\frac{m\pi z}{h}} + yz$

Table 2: Nondimentionalized fundamental frequencies of simply supported square plates $w^{-} = w\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}, a/h = 5.$

E_1/E_2	3D	Present	**Mantari 2011**	**Touratier 1991**	FSDT
3	6.618	6.5768	6.565	6.560	6.570
10	8.210	8.2675	8.286	8.274	8.298
20	9.560	9.5097	9.552	9.530	9.567
30	10.272	10.2464	10.305	10.277	10.326
40	10.752	10.7545	10.826	10.793	10.854

Table (3) Nondimentionalized fundamental frequencies a simply supported antisymmetric cross-ply square laminated plate with (a/h = 5), $w^{-} = w\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}$

E_1/E_2											
	Method	3	Diff %	10	Diff %	20	Diff %	30	Diff%	40	Diff%
$(0/90)_1$	3D Elasticity	6.257	-	6.984	-	7.674	-	8.176	-	8.562	-
	Kant 2002	6.156	1.161	6.936	0.681	7.688	0.206	8.257	0.990	8.709	1.176
	Raddy 1984	6.216	0,655	6.988	0.057	7.821	1.915	8.505	4.023	9.087	6.131
	S. Akavci 2008	6.218	0.623	6.993	0.281	7.832	6.058	8.522	4.231	9.111	6.412
	Present	6.220	0.591	6.981	0.042	7.819	2.557	8.506	4.036	9.091	6.178
$(0/90)_2$	3D Elasticity	6.545	-	8.144	-	9.405	-	10.165	-	10.679	-
	Kant 2002	6.431	1.741	8.101	0.527	9.433	0.291	10.246	0.796	10.799	1.123
	Raddy	6.500	0.687	8.195	0.626	9.626	2.349	10.534	3.630	11.171	4.607
	S. Akavci 2008	6.500	0.687	8.193	0.601	9.620	2.286	10.526	3.551	11.161	4.513
	Present	6.512	0.504	8.176	0.392	9.580	1.860	10.461	2.911	11.091	3.858
$(0/90)_3$	3D Elasticity	6.610	-	8.414	-	9.839	-	10.695	-	11.272	-
	Kant 2002	6.486	1.875	8.337	0.915	9.801	0.386	10.685	0.093	11.283	0.975
	Raddy	6.555	0.832	8.404	0.118	9.917	0.792	10.854	1.486	11.500	2.022
	S. Akavci 2008	6.556	0.816	8.405	0.106	9.918	0.830	10.856	1.505	11.503	2.049
	Present	6.570	0.605	8.388	0.309	9.876	0.376	10.797	0.953	11.433	1.428

Table (4) Nondimentional fundamental frequencies of square plate (0/90/0) at various thickness ratio(a/h), $w^{-} = w\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}$

	a/h		
Method	2	5	10
S.Xiang&K.Ming 2009	5.533	10.290	14.766
Raddy1997	5.205	10.698	14.753
Present	5.689	10.253	14.703

Table (5) Nondimentional fundamental frequencies for a four layer [0/90/90/0] for various thickness ratio (a/h), $\frac{E_1}{E_2} = 40\ , w^- = w\frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}$

a/h						
Method	5	10	20	25	50	100
Zhen&Wanji 2006	10.7294	15.1568	17.8035	18.2404	18.9022	19.1566
Wu et al.1994	10.6820	15.0690	17.6360	18.0550	18.6700	18.8350
Matsunaga 2000	10.6876	15.0271	17.6369	18.0557	18.6702	18.8352
Cho et al. 1991	10.6730	15.0660	17.5350	18.0540	18.6586	18.8350
Present	10.7544	15.1090	17.6590	18.0751	18.6854	18.8492

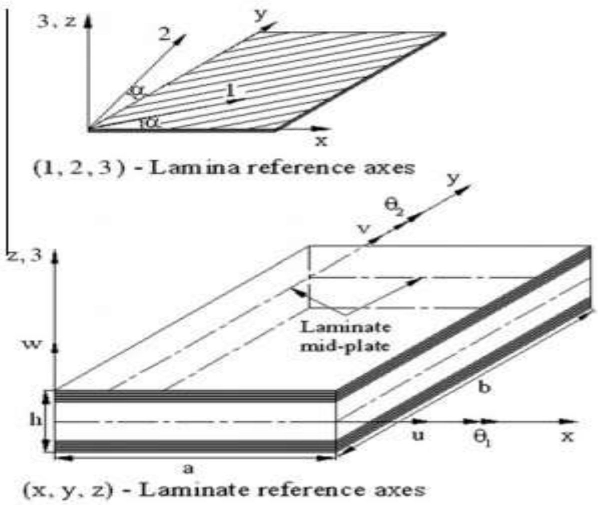

Fig.1: Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fiber orientation.

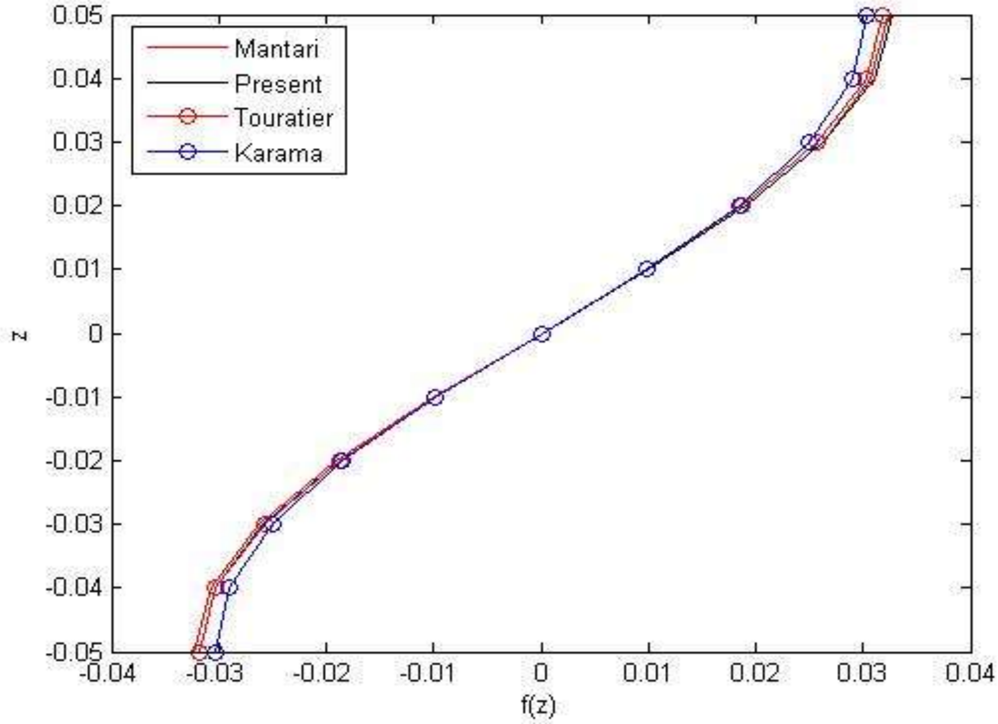

Fig .2: Shape strain functions of different shear deformation theories.

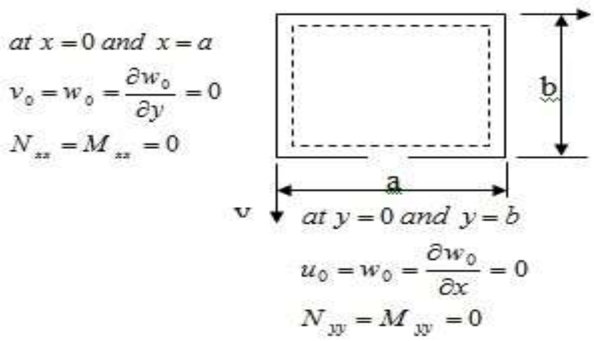

Fig.3: Boundary condition for simply supported plate .**J.N.Raddy 2003**

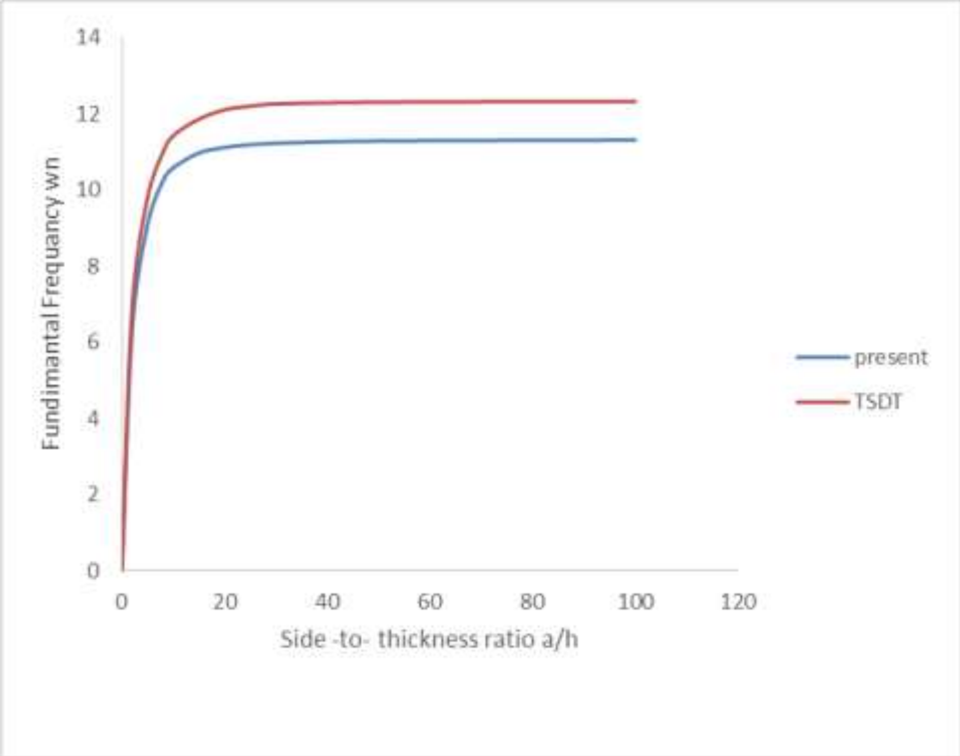

Fig.4: Nondimentionalized natural frequency versus side-to-thickness ratio (a/h) for cross-ply (0/90) at$\frac{E_1}{E_2} = 40$. Square plate .

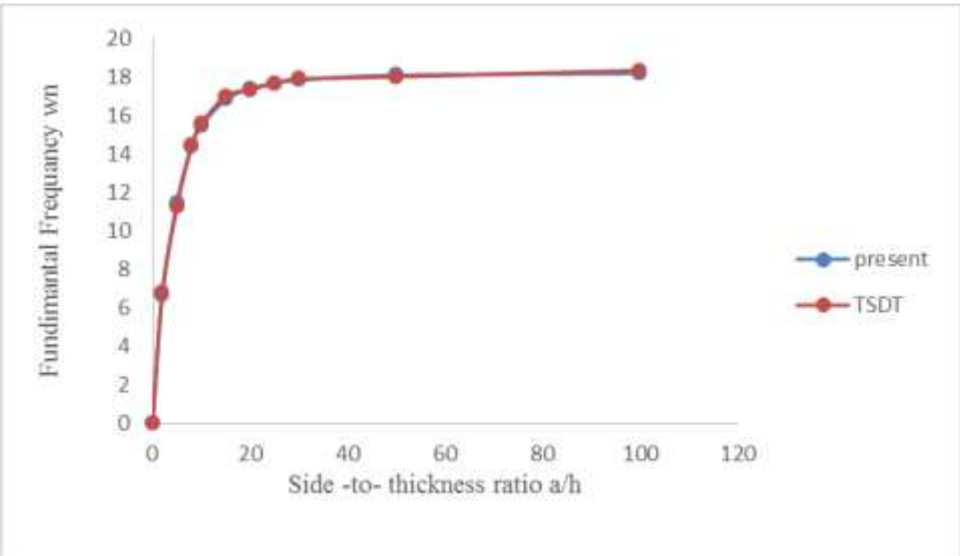

Fig.5: Nondimentionalized natural frequency versus side-to-thickness ratio (a/h) for cross-ply (0/90/0/90/0/90) at$\frac{E_1}{E_2} = 40$. Square plate.

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