EFFECT OF PITCH ANGLE ON STATIC CHARACTERISTICS OF A HELICAL MACHINED SPRING

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ABSTRACT :

This paper investigates the linear static and modal analysis of helical machined springs theoretically and numerically using the finite element method with the aid of SolidWorks software. A rectangular cross sectional area of a helical spring was studied with different pitch angle, it was found that increasing the pitch angle increases the spring deflection and decreases both the maximum shear stress and the fundamental natural frequency. Two cases were studied for machined spring, one with different pitch angle and the other with different number of spring starts. The machined helical spring showed better results for maximum shear stress and deflection compared with the conventional helical one. With multiple starts machined helical spring, the spring performance can be improved for a great extent. The results showed that SolidWorks software can be used effectively in modeling and analyzing machined spring.

Keywords: Helical spring, machined spring, pitch angle, finite element method, SolidWorks.

تأثير زاوية الخطوة على الخصائص الستاتيكية للنوابض الحلزونية المشغلة

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الخلاصة :

تبحث هذه الورقة التحليلات الخطية الستاتيكية وتحليلات الاهتزازات للنوابض الحلزونية المشغلة نظريا وعدديا باستخدام طريقة العناصر المحددة وبمساعدة البرنامج SolidWorks. تم دراسة نابض حلزوني ذو مقطع مربع وبزوايا خطوة مختلفة وقد وجد ان زيادة زاوية الخطوة تؤدي الى زيادة قيمة الانحناء ونقصان كلا من اجهاد القص الاقصى والاهتزاز الطبيعي الاساس. تم ايضا دراسة حالتين من النوابض المشغلة ، الاولى بزوايا خطوة مختلفة والاخرى لنابض ذو عدد بدايات مختلفة. النتائج الخاصة بإجهاد القص الاقصى والانحناء كانت افضل للنابض الحلزوني المشغل مقارنة مع النابض الحلزوني التقليدي. مع زيادة عدد بدايات النابض المشغل فان كفاءة الابض تتحسن بشكل كبير. اظهرت النتائج امكانية استخدام البرنامج SolidWorks بشكل فعال لنمذجة وتحليل النابض المشغل .

NOMENCLATURE

b = wire side length (mm).
C = spring index.
d = diameter of spring wire (mm)
D = spring mean diameter (mm).
E = Young's modulus (N/mm^2).
f = fundamental natural frequency (Hz)
G = shear modulus (N/mm^2).
K = Spring rate (N/mm)
k_1, k_2 = correction factors accounts for curvature and direct shear.
M = Spring mass (Kg)
n = No. of spring turns.
P = applied load (N).
α = pitch angle (degree)
ψ' = Constant
ξ, λ, μ, = Coefficients given by diagrams.
ψ = Stress coefficient dependant on ratio of wire side length and coil ratio given in diagram.
ε = Elasticity coefficient dependant on ratio of wire side length, given by diagram.
τ_{max} = maximum shear stress (N/mm^2)
δ_{max} = maximum deflection (mm)

1- INTRODUCTION :

Springs are flexible elements that can be used in different applications for exerting force, shock absorption and energy storage. they are commonly used in automobiles, watches, valves, scales, toys and many other applications. There are many different types of springs such as helical, spiral, leaf and conical springs; they are used in tension and compression. Helical compression springs are typically made from circular, square or rectangular cross section wire that is coiled in a helical shape. A constant pitch is normally maintained between the adjacent turns.

In die design the space occupied by the spring is an important factor in the design. The use of helical springs with rectangular cross section in the dies and especially in the cutting dies simplified the design process and makes it more economical and practical at the same time. **"**It is not recommended to use square or rectangular wire for springs unless space limitations make it necessary. These springs for a given space restriction are stronger than the round wire springs**"**, **[C. K. Vijayaraghavan, S. Vishnupriyan, 2009]** because their cross sectional area is greater than that of regular spring with circular cross section; this means larger mass of spring and in turn more force it can provide.

Machined springs meet the work requirements for small spaces and with great local capacity which cannot be provided by springs from other types. These springs have a possibility of obtaining very high rigidity and high dimensional accuracy, and thus reproducible stiffness. Threads, mounting flanges and closed attachments provide various ways of mounting the machined springs. The fixing portion is an integral part of the spring and therefore, they can work both compressive and tensile. Machined spring can be used as a flexible coupling for torque transmission, [**R. H. Williams, 1980**], or to compensate misalignment between the coupling's ends, [**A. O. Hunt, 1992**]. They can be used as a displacer in free-piston stirling engine to reduce working volume requirements, [**Pellizzari et, al 2006**]. Double start helical machined spring can be used as an electrically conductive member to complete the electric circuit of two components moving relative to each other, [**Dicken et al. 2013**].

[**E. ZEYDAN, 2006]** studied the vertical tip deflection of helical compression springs subjected to an axial force acting along the helix axis. He found that increasing the helix angle results an increase in tip deflection, while increasing the spring index decreases the tip deflection. [**J. Salwinski and K. Michalczyk, 2006]** used the Finite Element method for stress analysis in helical machined springs with closed ends coils, ; they concluded that decreasing the radius of curvature increases the stiffness of the spring. [**M. Taktak et al. 2008]** developed an analytical model for the dynamic analysis of a cylindrical isotropic helical spring to calculate the natural frequency and dynamic response of springs with circular, square and rectangular cross-sections. They used the hybrid-mixed formulation to compute the stiffness matrix. [**V. Brijpuria and K. K. Jain, 2013]** used ANSYS software for shape optimization of closed coil helical spring as well as for validation of final geometry in order to have minimum weight under design constraints. [**A. M. Choube and D. V. Bhope, 2013]** also used ANSYS to perform a stress and buckling analysis of helical spring with round, square and rectangular cross sections. [**Z. Kh. Poul and J. G. Nikam, 2013]** used the CAD software CATIA and UNI Graphics (UG) for fatigue analysis of a helical compression spring subjected to a cyclic loading.

This paper presents a theoretical and numerical investigation via SolidWorks software to predict the design characterization (maximum shear stress, maximum deflection and natural frequency) of the helical spring with rectangular cross section. The effect of spring pitch angle on maximum shear stress, maximum deflection and natural frequency for both conventional helical spring and machined helical springs with square cross section will be studied. The effect of number of starts of the helical machined spring on the above characters will be studied too.

2- THEORETICAL PART

2-1 Spring with square cross section

The maximum shear stress (τ_{max}) and maximum deflection (δ_{max}) for a helical compression spring with square cross section are **[Indian Standard, 1978]:**

$$\tau_{max} = \psi \frac{PD}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\delta_{max} = \frac{\varepsilon\, P\, D^3\, n}{G\, b^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

[R. S. Khurmi and J. K. Gupta, 2005] presented the following formulas to calculate the maximum shear stress (τ_{max}) and maximum deflection (δ_{max}):

$$\tau_{max} = k_1 \frac{2.4\, P\, D}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Where:

$$k_1 = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

And C = D/d

$$\delta_{max} = \frac{5.568\ P\ D^3\ n}{G\ b^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Equations (1 to 4) are not taking into account the effect of pitch angle (α), **[Wahl, 1944]** proposed equations (5 and 6) to represent the effect of pitch angle on maximum stress and maximum deflection for small pitch angles and indexes greater than three. The maximum shear stress is:

$$\tau_{max} = k_2 \frac{2.4\ P\ D \cos\alpha}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where:

$$k_2 = 1 + \frac{1.2}{C} + \frac{0.56}{C^2} + \frac{0.5}{C^3}$$

$$\delta_{max} = \psi' \frac{5.575\ P\ D^3\ n}{G\ b^4} \left(\frac{C^2 - 1}{C^2 - 0.69} \right) \quad \ldots\ldots\ldots (6)$$

$$\psi' = \frac{\cos\alpha}{1 + \frac{0.31 \cos^4\alpha}{C^2 - 1}} + 1.69 \frac{G}{E} \sin\alpha \tan\alpha$$

Springs can be used in rotating machines, so it is important to investigate their frequency analysis. For a helical spring with one end fixed and the other end free, the fundamental natural frequency can be calculated as follows **[J. Shigley, et al. 2004]:**

$$f = \frac{1}{4} \sqrt{\frac{\mathbf{K}}{\mathrm{M}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where:

$$\mathbf{K} = \frac{\xi\ G\ b^4}{\mathrm{n}\ D^3}$$

2-2 Machined square-coil springs

For machined helical springs with square cross section, the maximum shear stress and maximum deflection are **[R. O. Parmley, 2000]:**

$$\tau_{max} = \lambda \frac{2\ P\ (D + b)}{b^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\delta_{max} = \frac{P\ D^3\ (n - 1)}{\mu\ G\ b^4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

2- NUMERICAL PART :

Numerical simulation uses the numerical methods to represent the physical system or object; it can be used to help the designer to analyze, test, optimize and simulate the real-world situation of parts and structures before producing an expensive physical prototype. SolidWorks simulation is an add-in finite element analysis (FEA) application used to simulate the working conditions of the design and predict its behavior. It provides a friendly-user environment for virtual part design and prototyping. SolidWorks simulation can be used for static, modal, buckling, thermal and optimization simulation analysis **[J. E. Akin, 2009]**.

The typical analysis in SolidWorks consists of three stages: Pre-processor, Solution, and Post-processor. The pre-processor is the user interface environment that contains the required commands to build the model. The following basic dimensions are considered when modeling the helical spring with square cross section: mean diameter (40mm), wire cross section (10x10mm), number of active coils (12). Material properties are as follows: Steel (AISI-1045), Density =7850Kg/m^3, E = 205GPa, G = 80GPa, Yield strength = 530MPa. To obtain results convergence and minimize analysis time, the spring was meshed with the following properties, Solid mesh (4points), element size (3mm), this results in (83001) elements and (49988) nodes.

The solution processor provides the commands that are used to apply the specified boundary conditions and loads to the model. After providing the required parameters to the solution processor, it solves the model. The solution depends on the required type of analysis. In the present work STATIC and MODAL analysis are performed. As a case study and for static analysis, the spring is modeled as fixed at the lower end and a load of 100N is applied on the upper end. For modal analysis, the spring is modeled as fixed from both ends.

4- RESULTS AND DISCUSION :

4-1 Helical spring analysis

To study the effect of pitch angle on shear stress, deflection and natural frequency of helical spring with square cross section, different springs were modeled as shown in figure (1). Six different springs with pitch heights (12, 14, 16, 18, 20 and 22mm) were modeled to represent springs with different pitch angle (5.455, 6.357. 7.256, 8.152, 9.043 and 9.93^0) respectively.

The effect of pitch angle on maximum shear stress is shown in figure (2). It can be noted that the spring pitch angle has a slight effect on shear stress, as the pitch angle increases, the maximum shear stress decreases for both the analytical and numerical results. For a certain length of spring wire, increasing the pitch angle decreases the spring mean diameter value and this leads to the decrease in the shear stress too. Also it can be noted that as the pitch angle increases, the numerical result of the maximum shear stress converges to the theoretical results. The difference between the theoretical and analytical results is within the range (2.4% to 0.9%).

The effect of pitch angle on maximum deflection is shown in figure (3). It can be noted that as the pitch angle increases, the maximum deflection increases too for both the analytical and numerical results, this is due to the decrease in spring stiffness. According to equation (6), the deflection id directly related to the number of active coils. For a certain spring length, increasing the pitch angle decreases the number of active lengths and this in turns decreases the maximum deflection of the spring. The difference between the theoretical and analytical results is within the range (3% to 4.6%).

The numerical results for the fundamental natural frequency with the pitch angle are shown in figure (4). Again as the pitch angle increases, the fundamental natural frequency decreases due to the decrease n spring stiffness.

4-2 Machined spring analysis

To study the effect of pitch angle on shear stress, deflection and natural frequency of helical machined spring with square cross section, different springs were modeled as shown in figure (5). Six different springs with pitch angles (5.455, 6.357. 7.256, 8.152, 9.043 and 9.93^0) were used. The results are listed in table (1).

As the pitch angle increase the maximum shear stress decreases. For the same characteristics and range of pitch angle, the value of maximum shear stress is less than that of the helical spring and this indicates that the machined helical spring with square cross section is better than the conventional helical spring with square cross section.

As the pitch angle increases, there is a very small change in deflection, but there is a significant decrease in the fundamental natural frequency. Increasing the pitch leads to increasing the spring length which in turns increases the spring mass and decreases the fundamental frequency value.

To improve the design, another three helical machined springs with square cross section were modeled as shown in figure (6) to study the effect of number of starts on shear stress, deflection and natural frequency. The study results are listed in table (2). It can be noted that the maximum shear stress falls from 12.79N/mm^2 down to 3.61N/mm^2, and the maximum deflection falls from 0.468 to 0.049mm. Both these values indicate a great improvement in the spring design. For the fundamental natural frequency, again there is about 2.5 times improvement in its value.

It can be concluded that SolidWorks simulation can be used effectively to model and analyze both conventional helical spring and machined helical spring with square cross section. With multiple starts machined helical spring, the pitch angle can be increased without affecting the overall height of the spring; in turn we can improve the spring performance by decreasing the maximum shear stress, maximum deflection and increase the fundamental natural frequency.

Table 1- Effect of pitch angle on static characteristic of machined spring

Pitch angle deg.	τ_{max} N/mm^2	δ_{max} mm	f_n Hz
5.455	12.790	0.4677	196.13
6.357	12.759	0.4666	176.70
7.256	12.631	0.4656	159.50
8.152	12.413	0.4647	144.79
9.043	12.186	0.4636	132.19
9.930	12.141	0.4621	121.26

Table 2- Effect of spring starts on shear static characteristic of machined spring.

No. of starts	Pitch height mm	No. of turns	Pitch angle deg.	τ_{max} N/mm^2	δ_{max} mm	f_n Hz
1	12	12	5.455	12.790	0.4677	196.13
2	24	6	10.813	5.965	0.1129	365.88
3	36	4	15.986	3.610	0.0485	494.98

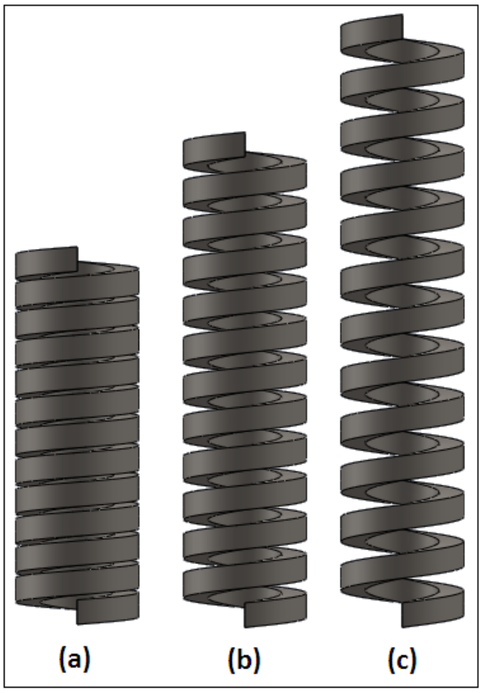

Figure 1: Helical spring with different pitch angle (degree)
a- 5.455, b- 7.256, c- 9.043

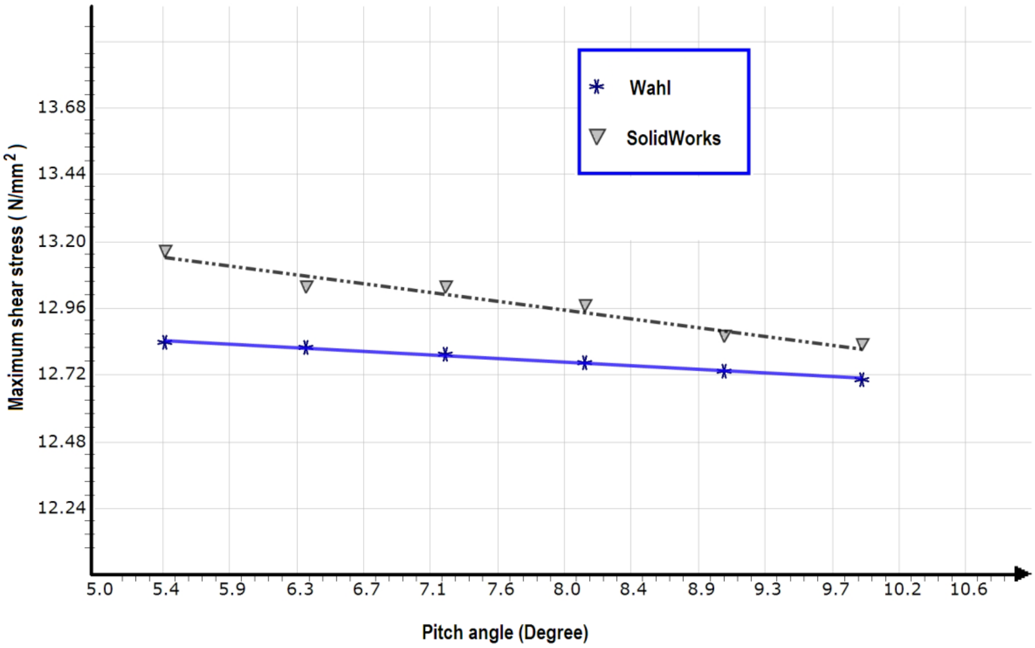

Figure 2: variation of maximum shear stress with pitch angle (helical spring).

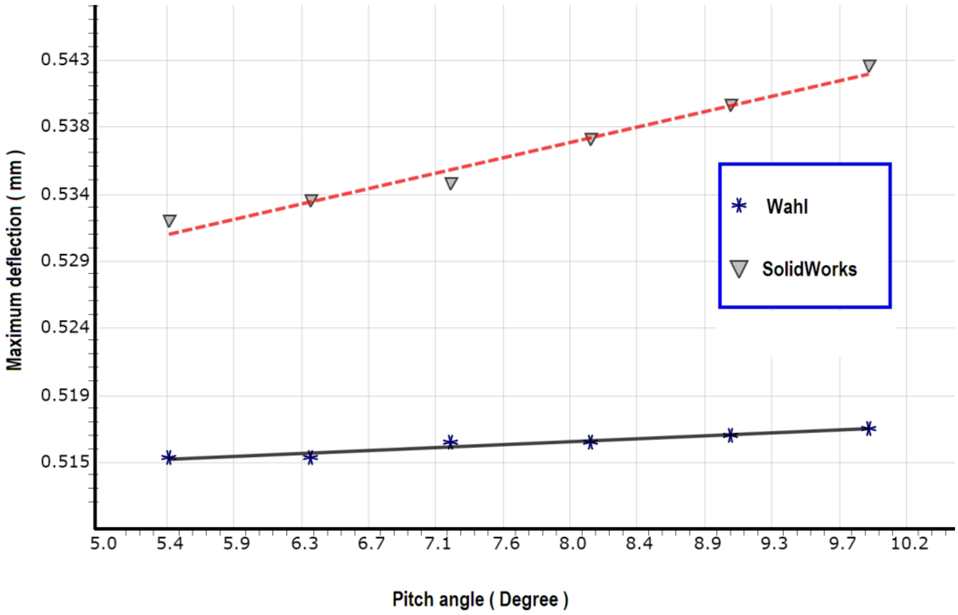

Figure 3: variation of maximum deflection with pitch angle (helical spring).

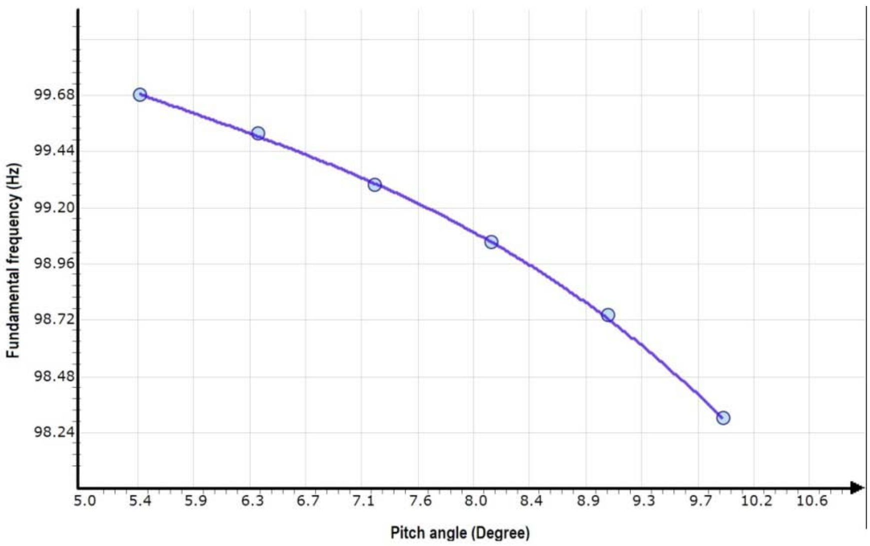

Figure 4: variation of maximum deflection with pitch angle (helical spring).

Figure 5: Machined spring with different pitch angle (degree)
a- 5.455, b- 6.357. c- 7.256

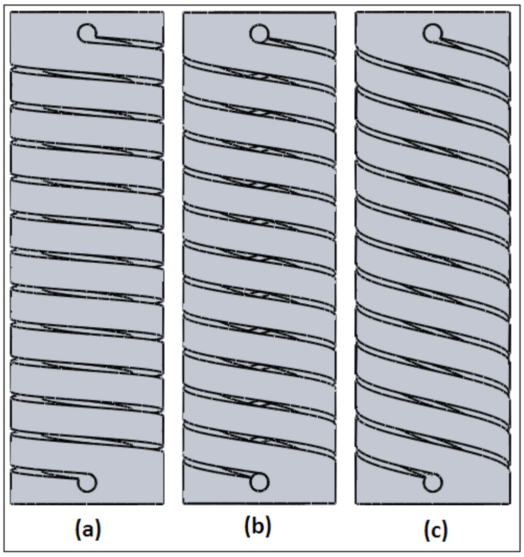

Figure 6: Machined spring with different spring starts.
a- One start b- Two starts c- Three starts

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